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AN EXPERIMENTAL STUDY OF YIELD SURFACES OF PRESTRAINED BRASS

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Abstract—Experimental results for twenty-eight tubular specimens of a M-63 brass, subjected to combined biaxial tension, are presented in the study of the effect of prestraining on the shape of the yield surface. Yield surfaces are found for various definitions of the yield locus. The strain increments are also shown in the stress plane in order to compare their orientation with the normality criterion. Moreover, the lengths of elastic and plastic strain increment vectors are compared at different moments of loading.

INTRODUCTION

The effect of prestraining on the shape of the yield surface was reported recently in several papers. Most of these experiments were performed using simultaneous loading of tubular thin-walled specimens by an axial force and a twisting moment. In the paper by Jagn and Shishmariev [1] all specimens were prestrained by an axial force beyond the initial yield locus, and next, after partial unloading, each of them was twisted using different loading paths. Each specimen has given one point only on the yield surface. In the work by Naghdi *et al.* [2] all specimens were subjected to prestraining by a twisting moment and next the shape of the yield surface was investigated. The proportional limit was taken as the yield locus. A similar loading path was used in the experiments by Ivey [3]. Bertsh and Findley [4] prestrained tubular specimens using various combinations of an axial force and a twisting moment. A particular mode of loading was used in experiments by Szczepiński [5]. Hu and Bratt [6] loaded tubular specimens simultaneously by a tension and an internal pressure, after prestraining by an axial force only.

In the present experiments tubular specimens were loaded by various combinations of an axial force and an internal pressure. Each of three groups of specimens has been prestrained in a different way. For two of these groups the end-point of their prestraining path was the same, but the prestraining histories were different.

SPECIMENS AND EQUIPMENT

The thin-walled tubular specimens were made of a drawn tube of 30 mm inner diameter and 1 mm wall thickness. The tube had its own deformation history. The material was M-63 brass containing 37 % of zinc. All specimens were annealed for 2 hr at a temperature of 650°C, cooled with the furnace for 20 hr until the temperature reached 200°C and then removed. Specimens were selected in order to obtain possibly uniform distribution of the wall thickness in longitudinal and circumferential directions. The largest deviation of the wall thickness from the mean value did not exceed 3 %.

The testing apparatus has two independent pressure installations. One of them gives the loading of the specimen by axial force and the other by circumferential stress only. The deformations have been measured by means of four ordinary resistance strain gauges with 15 mm gauge length, applied on the surface of each specimen, two in the axial and two in the circumferential direction at diametrically opposite positions at mid-length of the specimen. In order to eliminate possible slight deviations from symmetry, the strains were taken as the mean value of readings of two gauges oriented in the same manner. The measuring bridge used in the present work has the scale division corresponding to $\varepsilon = 5 \cdot 10^{-5}$.

EXPERIMENTAL RESULTS

Four sets of specimens, each of them containing seven specimens, were investigated, in order to establish the initial and three subsequent yield surfaces in the first quadrant of the axial stress (σ_z)–circumferential stress (σ_t) plane. In all experiments strains were recorded five minutes after the stress increment has been applied. Then the next stress increment was applied. For each specimen loaded along the prescribed loading path the equivalent stress–equivalent strain diagram was plotted, from which stresses corresponding to plastic equivalent strains $\varepsilon_i^p = 0{\cdot}01$, $0{\cdot}02$, $0{\cdot}1$, $0{\cdot}2$, $0{\cdot}3$, $0{\cdot}4$ and $0{\cdot}5\%$ were found. In this manner for each specimen seven points in the stress plane have been found. The experimental curves plotted through points corresponding to the same value of equivalent strain $\varepsilon_i^p = 0{\cdot}01, \ldots, 0{\cdot}5\%$ are further marked by subscript notation $\sigma_{0\cdot01}, \ldots \sigma_{0\cdot5}$, respectively. Moreover, the point of departure from the straight initial part of the diagram was determined. In this manner the proportional limit was obtained. It should be emphasized, however, that the value of the proportional limit depends notably on the accuracy of the extensometer and subjective estimate of the investigator. The elastic and plastic strain increments were obtained from the axial stress–axial strain and circumferential stress–circumferential strain diagrams.

Each of seven specimens belonging to the same set was prestrained in the same manner. Next, after unloading, the strain gauges were mounted on its surface, and two days later the yield locus of the specimen was investigated. The subsequent loading paths were different for each specimen of one set and were realized in a zigzag manner very close to the proportional loading. These zigzag loading paths were realized by subsequently applying small increments of axial force and internal pressure. The largest deviation from the radial path of proportional loading did not exceed 0·3 kg/mm^2.

In the first series of experiments the initial yield surface was found. Each of seven specimens with no prestrain was loaded along a certain radial path from the origin as shown in Fig. 1. The dashed lines show the proportional limit $\sigma_{\mathrm{prop}}^{(0)}$ and $\sigma_{0\cdot5}^{(0)}$ limit curves for the initial material. These curves are very close to the Huber–Mises ellipses.

In the second series each of the seven specimens was prestrained along the path OA (Fig. 1) beyond the initial yield locus and then unloaded. Next the curves corresponding to $\sigma_{\mathrm{prop}}^{1}$, $\sigma_{0\cdot01}^{1}, \ldots, \sigma_{0\cdot5}^{1}$ were found. None of these curves show agreement with either kinematic or isotropic strain-hardening hypothesis. In Fig. 1 vectors of elastic $d\varepsilon_e$ and plastic $d\varepsilon_p$ strain increments are also shown. In Fig. 1 and all further figures the elastic strain increment vectors are taken as equal to unity for comparison of their length with the length of corresponding plastic strain increment vectors. The length of plastic strain increment vectors enables us to observe the increase of the plastic part of

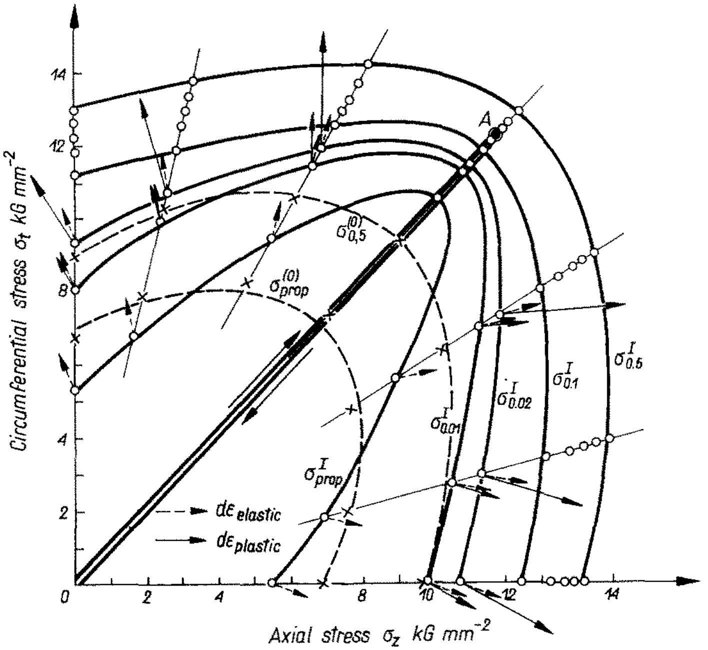

FIG. 1. Limit curves for the initial material (dashed lines) and for material prestrained until point A (continuous lines).

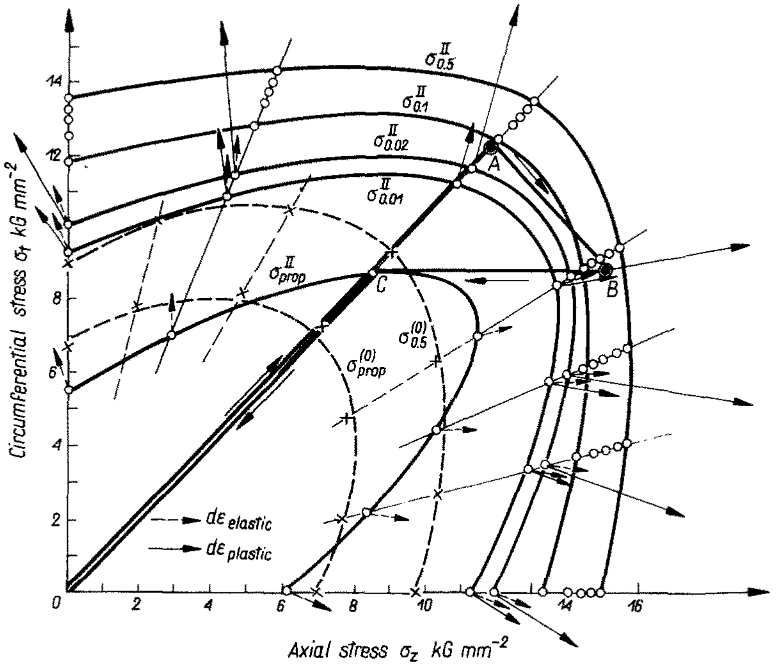

FIG. 2. Limit curves for the initial material (dashed lines) and for material prestrained along the loading path OAB (continuous lines).

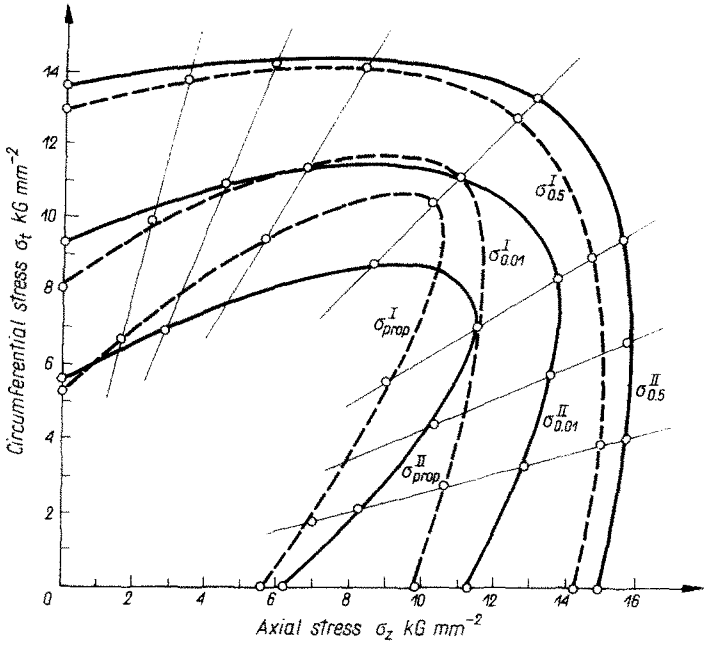

FIG. 3. Comparison of the limit curves from Figs. 1 and 2.

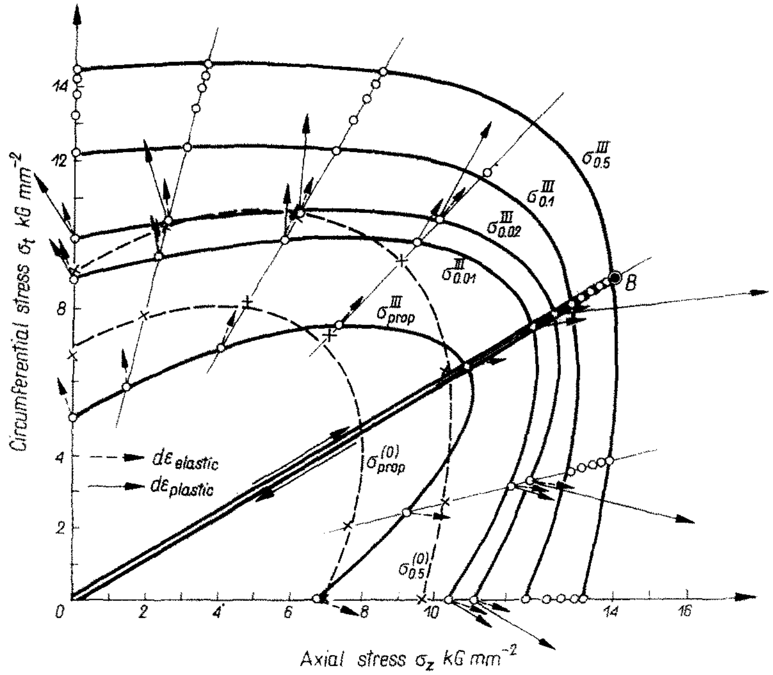

FIG. 4. Limit curves for the initial material (dashed lines) and for material prestrained until point B (continuous lines).

deformation. For proportional limit curve there is no plastic deformation. For the $\sigma^{I}_{0\cdot01}$-curve it is of the same order as the elastic part and for the $\sigma^{I}_{0\cdot01}$-curve its value is 3–4 times larger than the elastic part. For further curves this plastic part of deformation rapidly increases. One observes that plastic strain increment vectors are in general normal to corresponding curves, but at several points remarkable deviation from normality can also be observed. This deviation can be attributed to the fact that the limit curves were found using several specimens, which could have slightly different plastic behaviour.

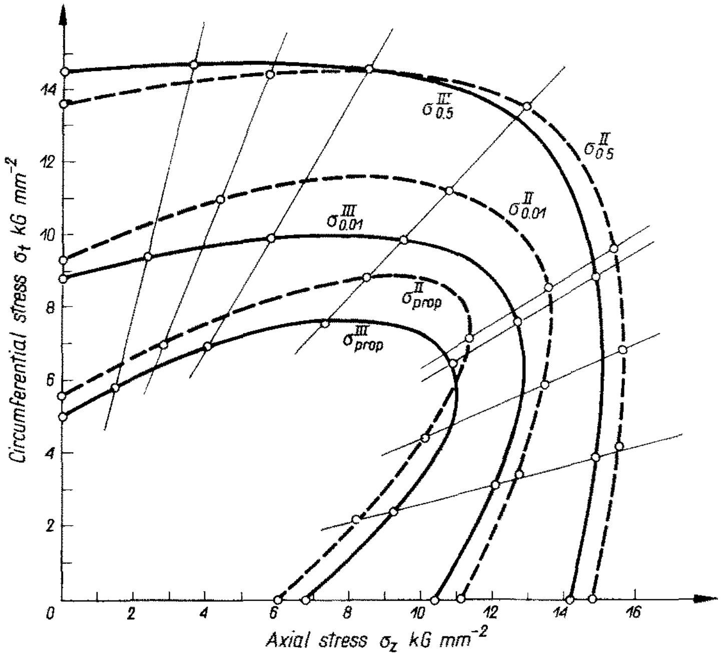

FIG. 5. Comparison of the limit curves from Figs. 2 and 4.

In the third series all specimens were prestrained first until the same point A as in the second series and next along the path AB (Fig. 2) and then unloaded along BCO. The yield surfaces of thus prestrained material were investigated in the same manner as before. Results are shown in Fig. 2. The σ_{prop}, $\sigma_{0\cdot01}$ and $\sigma_{0\cdot5}$ surfaces for the second (dashed lines) and third (continuous lines) series are compared in Fig. 3. There is a remarkable rotation of $\sigma^{II}_{\text{prop}}$-curve in respect to σ^{I}_{prop}-curve caused by additional sector AB of prestraining path. The $\sigma^{I}_{0\cdot5}$ and $\sigma^{II}_{0\cdot5}$-curves are, however, very close one to another.

In the fourth series all specimens were prestrained along the path OB (Fig. 4) and then unloaded. Although the end-point B is the same as in the third series the proportional limit curve $\sigma^{III}_{\text{prop}}$ is different than the $\sigma^{II}_{\text{prop}}$-curve in the third series. Both curves are compared in Fig. 5. Dashed lines represent yield curves for the third and continuous lines for the fourth series. Also in this case the difference between $\sigma_{0\cdot5}$-curves is very small.

FINAL REMARKS

The results presented above show strong effect of prestraining path on the proportional limit and $\sigma_{0 \cdot 01}$, $\sigma_{0 \cdot 02}$-curves. One of the interesting features of the proportional limit curve is its rotation shown in Fig. 3. Such rotation is not predicted by existing strain-hardening hypotheses. The $\sigma_{0 \cdot 5}$-curves depend very slightly on the prestraining path which is shown in Figs. 3 and 5. It was not the purpose of the present experiments to investigate the existence of the pointed corner in the yield surfaces. All experimental points are shown in figures, and corresponding limit curves were plotted through them. The decision whether the pointed corners exist or not would demand additional experimental points in the vicinity of the end-point of prestraining path.

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Zusammenfassung—In dieser Untersuchung, die darauf abzielt, den Einfluss einer Vordehnung auf die Gestalt der Fliessfläche zu ermitteln, werden die Versuchsergebnisse für 28 rohrförmige Probestücke aus M-63 Messing, welche einer kombinierten zweiachsialen Spannung unterzogen wurden, angegeben, und für verschiedene Definitionen des Fliesspannungsortes die Fliessflächen gefunden. Die Dehnungsinkremente werden auch in der Spannungsebene gezeigt, so dass deren Orientierung mit dem Normalitätskriterium verglichen werden kann. Ausserdem werden die Vektorlängen der elastischen und plastischen Dehnungsinkremente bei verschiedenen Beanspruchungsmomenten miteinander verglichen.

Абстракт—В работе приводятся результаты исследования влияния пластической деформации на форму поверхности текучести. Экперименты проведены на 28 тонкостенных трубчатых образцах из латуни М–63, о внутренним диаметром 30 мм и толщиной стенки 1 мм.

Испытаниа проводились на специальном устройстве, в котором две независимые гидравлические системы дают возможност одновременно нагружать образцы осевой силой и внутренним давлением.

В каждой из серий все образцы одинаково пластически деформировались но по разному для каждой серии. Далее различными способами нагружения каждого из образцов исследовалась форма поверхности текучести. Найдены поверхности соответствуюшие разным определениям поверхности текучести.

Одновременно определены направления приращения пластической деформации для проверки их перпендикулярности поверхности текучести.

Найденные поверхности текучести характерны значительным влиянием предварительной деформации на их форму. Особенно это относится к поверхностям σ_{npon}, $\sigma_{0 \cdot 01}$, $\sigma_{0 \cdot 02}$.

Поверхность соответствуюшая пределу пропорциональности в некоторых случаях поварачивается.